**Math 10B with Professor Stankova**
**Quiz 7; Tuesday, 3/12/2019**
**Section #203; Time: 11 AM**
**GSI name: Roy Zhao** **Name:** ____________________

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Circle True or False or leave blank. (1 point for correct answer, $-1$ for incorrect answer, 0 if left blank)

1. True **FALSE** The geometric distribution assumes that trials are dependent (without replacement) while the binomial distribution assumes that trials are independent.

> **Solution:** The geometric and binomial distribution both assume that the trials are independent.

2. **TRUE** False If $f$ is the PMF of a random variable $X$, it is possible for $f(E[X]) = 0$.

> **Solution:** An example is a die roll. The expected value of a die roll is 3.5 but it is not possible to roll a 3.5.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (3 points) I am fishing and the number of spots each fish has is Poisson distributed with an average of 0.5 spots per fish. What is the probability that in a group of 12 fish, they have a total of 8 spots total?

> **Solution:** In a group of 12, we expect to see $\lambda = 12 \cdot 0.5 = 6$ spots amongst them. This is Poisson distributed so the probability of them having 8 spots among them is $f(8) = \frac{\lambda^8 e^{-\lambda}}{8!} = \frac{6^8 e^{-6}}{8!}$.

(b) (4 points) Now suppose I catch 30 random fish and on average, I expect that 10 are striped. If there are 100 total striped fish in the pool, how many total fish are there?

**Solution:** This is a hyper-geometric distribution because out of the total $N$ fish, there are $m = 100$ who are striped and in a selection of $n = 30$ fish, we expect to see $E[X] = 10$ of them. Thus, we have that

$$10 = E[X] = \frac{mn}{N} = \frac{100 \cdot 30}{N}.$$

So $N = \frac{100 \cdot 30}{10} = 300$.

(c) (3 points) With the same numbers as part (b), suppose that I catch 10 fish. What is the probability that amongst them, 3 of them have stripes?

**Solution:** As said before, this is a hyper-geometric distribution with $m = 100$ and $N = 300$. Then $n = 10$ because there are 10 fish picked and we want to calculate the probability that we have $k = 4$ striped ones. The probability of this is

$$f(4) = \frac{\binom{m}{k}\binom{N-m}{n-k}}{\binom{N}{n}} = \frac{\binom{100}{3}\binom{200}{7}}{\binom{300}{10}}.$$